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The Tensile Strength of Hybrid Fibre Composites: A Probabilistic Analysis of the Hybrid Effects

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The tensile strength of hybrid fibre composites: a probabilistic analysis of the hybrid effects

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Contents

A hybrid composite formed by brittle and less brittle fibres embedded into a ductile matrix is taken as the model for study. The breaking strains of distinct fibre types in the hybrid composite are considered in this paper as statistical variables so that the stress–strain relationship of the composite is treated as a typical stochastic process indexed with the composite strain. Also the mechanism reflecting the interactions between the constituents revealed by the fragmentation process during composite extension, and the effect of breaking strain variation between individual fibres of the same type have been included in the stochastic formula. Based on this theoretical model, the prediction of the entire stress–strain curve of a hybrid composite is provided in this study. A comprehensive parametric study is then carried out to illustrate the inter-relationships between the composite breaking behaviour and the fibre and composite interfacial properties.

Next, as the hybrid effect is defined by comparing the hybrid composite either with the rule of mixtures using single fibre properties or fibre bundle properties, or with a non-hybrid composite, the differences in these cases are discussed in detail. Various sources causing the hybrid effect are revealed, studied and summarized.

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These sources are found due partly to the property variations of the fibres which affect the interactions in the hybrid composite between fibres and matrix, and partly to the cross coupling interactions between the fibres of different types in the hybrid composite. The impacts of these sources on the composite behaviour as well as the associated hybrid effects are demonstrated and the important factors are detected and examined. It is then stressed in this paper that since most of the mechanisms responsible for causing the so-called hybrid effects have been proved effective in non-hybrid composites as well, we need to be more cautious concerning the definition for the hybrid effects in a hybrid composite.

Finally, the relationships between the hybrid effects and the important variables such as the relative hybrid ratio, a_{LE}, the relative fibre–matrix interface shear strength $\tau_{\mathrm{y}}/E_{\mathrm{f}}$, and the relative fibre tensile modulus $E_{\mathrm{fHE}}/E_{\mathrm{fLE}}$ are investigated and illustrated.

1. Introduction

Hybrid fibre composites are formed by impregnating two or more types of fibres with different mechanical properties into a common matrix. This kind of composite provides a wider range of system properties, performance compensation between fibres of different types, and an effective reduction of cost, making hybrid composites increasingly popular in various applications. The study of hybrid composite behaviour, especially their mechanical properties, has been an active area in composite material research in the past few decades.

The initial approach to predict the mechanical properties of a hybrid composite has been the adoption of the simple averaging law better known as the rule of mixtures. Although some of the system properties, typically the tensile elastic modulus, obey the rule of mixtures, i.e. the system property being the average value of the constituents weighted by their respective proportions, many other predictions from this rule are not satisfactory. System properties such as the breaking stress and strain have been found to deviate considerably from the results based on the rule of mixtures. These deviations of behaviour of a hybrid structure from the rule of mixtures have been termed the 'hybrid effects' (Marom *et al.* 1978; Summerscales & Short 1978). A positive hybrid effect means that the property is above the prediction given by the rule of mixtures, whereas a negative hybrid effect means the property is below the prediction.

An alternative definition for the hybrid effect is the difference between the performance of a fibre in a hybrid composite and in a monolithic or single fibre composite (Bader & Manders 1981*a*; Fukuda *et al.* 1984*b*; Harlow 1983). In this study, we will compare these two definitions to show their applicability to different cases.

There have been numerous studies on the mechanical behaviour of hybrid composites theoretically and experimentally, and some of the representative papers are listed as references in this article (Aveston & Sillwood 1976; Aveston & Kelly 1980; Bader & Manders 1981*a*, *b*; Bunsell & Harris 1974; Chou 1992; Fariborz *et al.* 1985; Fukuda 1983; Fukuda & Chou 1983; Fukuda *et al.* 1984*a*, *b*; Fukunaga & Chou 1984; Fukunaga *et al.* 1984, 1989; Gruber & Chou 1983; Harlow 1983; Kretsis 1987; Kirk *et al.* 1978; Manders & Bader 1981*a*, *b*; Marom *et al.* 1978; Marom *et al.* 1978; Phillips 1976; Piggot & Harris 1981; Pitkethly & Bader 1987; Qiu & Schwartz 1993*a*, *b*; Rosen

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1965; Summerscales & Short 1978; Takahashi & Chou 1987; Wagner & Marom 1982; Xing *et al.* 1981; Yau & Chou 1989; Zweben 1970, 1977). In the investigations so far, the statistical variations in fibre properties (Fukunaga *et al.* 1984), and the fibre dispersion and hence the load sharing pattern (Fariborz *et al.* 1985) are held partially responsible for the hybrid effects, but the exact mechanisms have remained somewhat of a mystery. There are conflicting experimental and theoretical results reported in terms of the nature of the hybrid effects. For instance, Zweben (1977) predicted that whether a positive or a negative hybrid effect occurs is dependent mainly on the ratio of the fibre extensional stiffness or modulus. Harlow (1983) applied a probabilistic model of a chain-of-bundles for unidirectional, intraply hybrid composites and demonstrated in his model that there was a negative hybrid effect for the breaking strain, but a positive hybrid effect for the tensile strength. Aveston & Sillwood (1976) reported, in their study on a carbon–glass–epoxy composite, a positive hybrid effect for both the failure strain and failure energy (hence the strength). Yet most recently, Qiu & Schwartz (1993*a*) have stated that a negative hybrid effect was observed for the tensile strength of a Kevlar-149–S-glass hybrid composite, and also the breaking strain of the Kevlar fibres was found to be higher in the hybrid composite than in a composite with Kevlar fibres alone.

As to the causes responsible for the hybrid effects, Fukunaga *et al.* (1984) claimed that the hybrid effects result from the scattering of fibre strength; the size of the fibre bundle and the state of stress redistribution have the dominant influence on the magnitude of the hybrid effects. The stress concentration and the dynamic effects are also considered accountable for causi ng the hybrid effects (Chou 1992). Qiu & Schwartz (1993*a*) have summarized three possible causes for the hybrid effects: (1) thermal residual stress (Bunsell & Harris 1974), (2) the role of higher elongation fibres as crack arresters (Xing *et al.* 1981), and (3) smaller dynamic stress concentration factors (Fukunaga & Chou 1984).

Understanding of the mechanisms leading to the hybrid effects is the crucial link towards the understanding of the mechanical behaviour of hybrid composites. Once we can identify the related factors, a modified version of the rule of mixtures capable of predicting the hybrid effects will provide a powerful technique for hybrid composite design.

In the present paper, the stress–strain relation of a hybrid composite is treated as a stochastic process with the composite strain as the independent variable. Incorporating other important factors, we are able to predict the entire stress–strain curve for a hybrid composite, and using the above two different definitions for the hybrid effects, we will expose various factors causing the hybrid effects.

The following assumptions are made in the present work.

(i) Fibre breaking strain distribution is of Weibull form.

(ii) Fibres have a linear stress–strain relationship up to breakage.

(iii) The changes of the interfacial properties between fibre and matrix during composite extension before failure are negligible.

(iv) There is an intimate blending and uniform distribution of two fibre types in the hybrid composite.

(v) When a fibre breaks, the load it was carrying is equally shared among the surviving fibres. The effects of fibre dispersion (Fariborz *et al.* 1985), stress concentration and the dynamic wave propagation (Fukunaga & Chou 1984) are ignored. In this work, we have utilized the results of Daniels's analysis (Daniels 1945) on fibre bundles, and the equal load sharing assumption was adopted in his analysis.

Yet, in a practical composite, local load sharing of various forms is more appropriate. Therefore, conclusions from this work can be refined by including more complex load sharing patterns.

For easy manipulation, the composite strain, instead of the stress, has been taken as the independent variable in studying the hybrid effects. However, because of the direct relation between the two, the conclusions based on the breaking strain can always be readily extended to composite strength.

Furthermore, to present the essence of this study clearly, in the following analysis we only use the most widely accepted, rather than the latest, theoretical results available on the relevant topics. Our study can surely be improved by substituting in the most advanced theories.

2. Strength of a hybrid composite system

A hybrid composite of a unidirectional lamina formed by a brittle and a less brittle fibre embedded into a ductile matrix is taken as the model for study. The two fibre types are designated as LE and HE (low and high elongations, respectively), and are continuous and arranged parallel to the loading direction.

If V_{f} is the total fibre volume fraction of the composite, the fractions for the two fibre types will be $a_{\mathrm{LE}}V_{\mathrm{f}}$ for type LE and $a_{\mathrm{HE}}V_{\mathrm{f}}$ for type HE respectively where the fraction coefficients $a_{\mathrm{LE}} + a_{\mathrm{HE}} = 1$.

When there is a tensile strain ϵ_{c} applied to the composite, the stress σ_{c} on the hybrid composite is then the sum of the contributions of the three constituents and, according to the rule of mixtures, can be expressed as

$$\begin{aligned}\sigma_{\mathrm{c}} &= (a_{\mathrm{LE}}\sigma_{\mathrm{LE}} + a_{\mathrm{HE}}\sigma_{\mathrm{HE}})V_{\mathrm{f}} + (1 - V_{\mathrm{f}})\sigma_{\mathrm{m}} \\ &= [(a_{\mathrm{LE}}E_{\mathrm{fLE}} + a_{\mathrm{HE}}E_{\mathrm{fHE}})V_{\mathrm{f}} + (1 - V_{\mathrm{f}})E_{\mathrm{m}}]\epsilon_{\mathrm{c}}, \qquad (2.1)\end{aligned}$$

where σ_i and E_i are the stress and tensile modulus for the constituent i, and i = LE, HE, m, representing the LE fibre, the HE fibre and the matrix, respectively. It will be proven later that the tensile modulus of a fibre bundle is identical to the tensile modulus of the fibre.

However, enough experimental evidence (Aveston & Sillwood 1976; Bunsell & Harris 1974; Fukunaga & Chou 1984; Fukunaga *et al.* 1984; Harlow 1983; Qiu & Schwartz 1993*a*; Zweben 1977) has demonstrated that the actual result deviates from this prediction, especially when the composite strain is close to the breaking strains of the fibres. At this point, two possible causes can be considered responsible for this deviation.

First of all, the fibres of the same type are not uniform in properties, and when they form a fibre bundle embedded into the matrix, this non-uniformity, as expected, will lead to dispersions in the properties of the fibre bundle. In other words, properties like the breaking strain of the fibre bundle will obey a statistical distribution instead of being a single constant value. Consequently, when composite strain increases to a level high enough to cause fibre failure, the fibres of the same type will not break simultaneously; there is a range of composite strain for fibres of the same type to fail completely. To account for this effect, equation (2.1) has to be modified into

$$\sigma_{\mathrm{c}} = [(a_{\mathrm{LE}}S_{\mathrm{LE}}E_{\mathrm{fLE}} + a_{\mathrm{HE}}S_{\mathrm{HE}}E_{\mathrm{fHE}})V_{\mathrm{f}} + (1 - V_{\mathrm{f}})E_{\mathrm{m}}]\epsilon_{\mathrm{c}}, \qquad (2.2)$$

where S_{LE} and S_{HE} correspond to the ratios of the number of fibre type LE and HE which are not broken yet and are still carrying the load. These two surviving fibre ratios can be derived as

$$S_i = \int_{\epsilon_c}^{\infty} H_i(\epsilon_b)\, d\epsilon_b, \tag{2.3}$$

where $H_i(\epsilon_b)$ is the distribution density function of the breaking strain ϵ_b of the fibre bundle type i, and $i =$ LE and HE.

It is now clear that the stress (and strength) of the composite is not deterministic and is in fact subject to uncertainty stemmed from the non-uniformity of the breaking strains between fibres of the same type. In other words, it is more realistic to treat the composite strength as a stochastic process indexed with its strain ϵ_c. The composite tensile modulus can then be derived in relative terms from equation (2.2) as

$$\frac{E_c}{E_m} = \left(a_{LE} S_{LE} \frac{E_{fLE}}{E_m} + a_{HE} S_{HE} \frac{E_{fHE}}{E_m} \right) V_f + (1 - V_f). \tag{2.4}$$

The second problem which exists in equation (2.1) is the lack of consideration of the interactions between the constituents in a composite under extension. It has become known (Kelly & Macmillan 1986) that the interaction between the constituents in a fibrous structure will considerably alter their behaviour and their contributions to system strength. A more reasonable prediction will follow if we can include the effects of this interaction into equation (2.2).

In the following two sections, the form of the breaking strain distribution function $H_i(\epsilon_b)$ for fibre bundles is provided and the effects of the *in situ* interactions are introduced.

3. Breaking strain distribution function for fibre bundles

As the structure of fibres is not uniform, the breaking strain tested from the specimens of a fibre type will hence be a statistical variable. Assume its cumulative probability distribution function is of Weibull type. So for a fibre with length l_f, the probability of the fibre breaking strain being ϵ_f can be written as

$$F(\epsilon_f) = 1 - \exp[-l_f \kappa \epsilon_f^{\lambda}], \tag{3.1}$$

where κ is the scale parameter and λ is the shape parameter of the fibre.

It has to be pointed out that the Weibull function has several slightly different versions (Harlow 1983; Zweben 1977), we here use equation (3.1) as in Zweben (1977) for its easy manipulation.

The mean or expected fibre breaking strain $\overline{\epsilon_f}$ can then be calculated as

$$\overline{\epsilon_f} = (l_f \kappa)^{-1/\lambda} \Gamma(1 + 1/\lambda), \tag{3.2}$$

where Γ is the gamma function, and the standard deviation of the breaking strain is

$$\Theta_f = \overline{\epsilon_f} \left[\frac{\Gamma(1 + 2/\lambda)}{\Gamma^2(1 + 1/\lambda)} - 1 \right]^{1/2}. \tag{3.3}$$

Let us then consider a fibrous system of N fibres forming a parallel bundle with no interaction between individual fibres. Following Daniels's (1945) analysis, for a large bundle of high N value, the density distribution function of the bundle breaking

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strain ϵ_b would approach a normal form,

$$H(\epsilon_\mathrm{b}) = \frac{1}{\sqrt{2\pi}\,\Theta_\mathrm{p}} \exp\left[-\frac{(\epsilon_\mathrm{b} - \overline{\epsilon_\mathrm{b}})^2}{2\Theta_\mathrm{p}^2}\right], \tag{3.4}$$

where $\overline{\epsilon_\mathrm{b}}$ is the expected value of the bundle breaking strain,

$$\overline{\epsilon_\mathrm{b}} = (l_\mathrm{f}\kappa\lambda)^{-1/\lambda}\exp(-1/\lambda) \tag{3.5}$$

and Θ_p is the standard deviation of the breaking strain

$$\Theta_\mathrm{p}^2 = (l_\mathrm{f}\kappa\lambda)^{-2/\lambda}[\exp(-1/\lambda)][1 - \exp(-1/\lambda)]N^{-1}. \tag{3.6}$$

It can be readily shown from these equations that both the breaking strain and its variation for a fibre bundle are smaller than those of the fibres. These reductions are caused by the breaking strain variation between the individual fibres forming the bundle. Since the modulus of this fibre bundle can be calculated from the modulus of the fibres based on the rule of mixtures, it is thus clear that the tensile modulus of the fibre bundle is identical to that of the fibre.

4. The interaction between fibres and matrix in a composite

The previous research findings, for instance in Kelly & Macmillan (1986), suggest that fibres, once embedded into a matrix material, will behave differently due to the interaction between fibres and the matrix. This interaction will inevitably alter the properties of the fibres.

During the fracture process of both composites (Harlow 1983; Zweben 1977) and textile yarns (Monego & Backer 1968; Pan 1993), it has been frequently observed that, as long as there is a difference between the breaking strains of the constituents, each individual fibre will break repeatedly with increasing strain of the structure before overall system failure. This phenomenon is known as the fragmentation process, a process in which a broken fibre can again build up tension, carry load, break into even shorter segments and still contribute towards overall system strength until the length of the fibre fragments reaches a minimum value where load can no longer build up to its breaking strength. This length is well known as the critical length, and the fragmentation phenomenon is said to have reached its saturation state. If $\overline{\sigma_\mathrm{f}}$ is the tensile stress which causes the fibre to break, then under the saturated situation, this critical length l_c is given as (Kelly & Macmillan 1986)

$$l_\mathrm{c} = \frac{r_\mathrm{f}\overline{\sigma_\mathrm{f}}}{\tau_\mathrm{y}} = \frac{r_\mathrm{f}E_\mathrm{f}\overline{\epsilon_\mathrm{f}}}{\tau_\mathrm{y}}, \tag{4.1}$$

where r_f is the fibre radius and τ_y is the yielding shear strength of the matrix adjacent to the interface or that of the fibre–matrix interface, whichever is less.

During the composite extension, by definition any fibre fragment with length longer than l_c is still able to break somewhere along its centre section as its strain exceeds its current breaking strain $\overline{\epsilon_\mathrm{f}}$. So the subsequent lengths of the fragments actually vary in the range of $\frac{1}{2}l_\mathrm{c}$ to l_c, with the mean length being $\frac{3}{4}l_\mathrm{c}$. Therefore, the mean length before fibres break into l_c will be $\frac{4}{3}l_\mathrm{c}$, and this length will be used to determine the current mean breaking strain $\overline{\epsilon_\mathrm{f}}$ from equation (3.2) for the new fibre fragments. Keeping this in mind and combining equations (3.2) and (4.1) gives the critical length as

$$l_c = \left[\frac{r_f E_f(\frac{4}{3}\kappa)^{-1/\lambda}\Gamma(1+1/\lambda)}{\tau_y}\right]^{\lambda/(1+\lambda)}. \tag{4.2}$$

As stated previously, we realize that the definition of the critical fibre length in equation (4.2) is a relatively simple result, and there have been several more complex models proposed, for instance in Feillard *et al.* (1994). Yet for briefness, we still apply equation (4.2) in our analysis.

The *in situ* fibre breaking strain $\overline{\epsilon_f}$ can then be determined based on equation (3.2) replacing the original fibre length l_f by the critical fibre length l_c of equation (4.2).

A complete and more realistic stress–strain curve and the composite strengths and breaking strains can then be predicted by combining the interaction effect represented by the critical fibre length l_c and the fibre variation influence specified by the surviving fibre ratio S into equation (2.2).

5. Determination of the breaking strains of a composite

Keeping in mind the discrepancies in properties between a fibre and a fibre bundle, we have to consider the fibres in the composite as fibre bundles rather than individual fibres. In fact, the breaking strain of a composite is not identical to that of either its constituent fibre bundle or the matrix. Determination of the breaking strain of a composite is complicated mainly for two reasons: first, the fibres of the same type in a bundle do not break simultaneously in the composite because of the variation of their breaking strains; and also the *in situ* breaking strain of a fibre will be altered from its original value due to the fibre–matrix interactions revealed by the fragmentation phenomenon. Therefore, even for a non-hybrid composite, we still have to adopt a special technique to find its actual breaking strain. Whereas for a hybrid composite, since the LE and HE bundles do not break at the same strain level, we have to examine the two breaking strains and strengths separately.

Moreover, the stress–strain curve of a hybrid fibre composite in general shows two peaks corresponding to the two fibre types, which will be named here for convenience as the principal and the secondary peaks. If there were no fibre–matrix interactions or no cross coupling effects between the LE and HE fibres in the hybrid composite, the principal and the secondary peaks would be identical to the peaks for monolithic composites made of single fibre type LE and HE respectively. So compared to the problem in a single fibre composite, it is even more difficult to derive the actual breaking strains in a hybrid composite because of the possible cross coupling effects between the two fibre types.

A rational definition for the *in situ* breaking strains of the fibre bundles should be the strains corresponding to the maximum composite stresses or the peak loads. In other words, the *in situ* breaking strains of the fibre bundles are those that lead to maximum values of the composite stress defined in equation (2.2). So the values of the breaking strains can be found from the solutions of the equation,

$$\frac{d\sigma_c}{d\epsilon_c} = \frac{d[(a_{LE}S_{LE}E_{fLE} + a_{HE}S_{HE}E_{fHE})V_f + (1-V_f)E_m]\epsilon_c}{d\epsilon_c} = 0. \tag{5.1}$$

Because of the assumption of linearity of the constituents, E_{fLE}, E_{fHE} and E_m will

Table 1. *Fibre and matrix properties*

item	fibre LE	fibre HE	matrix	unit
fibre radius r_{f}	5×10^{-3}	5×10^{-3}		mm
fibre length l_{f}	10	10		mm
relative tensile modulus	50	20	1	GPa
fibre mean breaking strain $\overline{\epsilon_{\mathrm{f}}}$	2.591	5.819		(%)
fibre mean strength $\overline{\sigma_{\mathrm{f}}}$	1.866	1.676		GPa
fibre number N_i	$200\times a_{\mathrm{LE}}$	$200\times a_{\mathrm{HE}}$		
fibre shape parameter λ	7	9		
fibre scale parameter κ	8.0×10^{9}	8.0×10^{9}		$(\mathrm{mm\ GPa}^{\lambda})^{-1}$
relative shear yielding stress $\tau_{\mathrm{y}}/E_{\mathrm{f}}$	0.02	0.02		

be independent of the composite strain ϵ_{c}. So the equation can be changed to

$$\frac{\mathrm{d}\sigma_{\mathrm{c}}}{\mathrm{d}\epsilon_{\mathrm{c}}} = a_{\mathrm{LE}}E_{\mathrm{fLE}}\frac{\mathrm{d}(S_{\mathrm{LE}}\epsilon_{\mathrm{c}})}{\mathrm{d}\epsilon_{\mathrm{c}}} + a_{\mathrm{HE}}E_{\mathrm{fHE}}\frac{\mathrm{d}(S_{\mathrm{LE}}\epsilon_{\mathrm{c}})}{\mathrm{d}\epsilon_{\mathrm{c}}} + (1-V_{\mathrm{f}})E_{\mathrm{m}} = 0. \tag{5.2}$$

The derivative $\mathrm{d}(S_i\epsilon_{\mathrm{c}})/\mathrm{d}\epsilon_{\mathrm{c}}$ can be found based on the calculus theory as

$$\frac{\mathrm{d}(S_i\epsilon_{\mathrm{c}})}{\mathrm{d}\epsilon_{\mathrm{c}}} = \epsilon_{\mathrm{c}}\frac{\mathrm{d}S_i}{\mathrm{d}\epsilon_{\mathrm{c}}} + S_i = -\epsilon_{\mathrm{c}}H_i(\epsilon_{\mathrm{c}}) + S_i, \tag{5.3}$$

where $H_i(\epsilon_{\mathrm{c}})$ is obtained from equations (3.4)–(3.6) as

$$H_i(\epsilon_{\mathrm{c}}) = \frac{1}{\sqrt{2\pi}\,\Theta_{\mathrm{b}}}\exp\left[-\frac{(\epsilon_{\mathrm{c}}-\overline{\epsilon_{\mathrm{b}}})^2}{2\Theta_{\mathrm{b}}^2}\right]. \tag{5.4}$$

Clearly, it is hard to derive a closed form for the solutions of equation (5.2). The numerical results are therefore obtained. Equation (5.2) in fact has several solutions. The first solution corresponds to the principal breaking strain, and the secondary breaking strain of the composite has to be determined in comparison with the stress–strain curve of the composite. Once a breaking strain is determined, the corresponding breaking strength of the composite can be calculated from equation (2.2).

6. Predictions and discussion

The assumed fibre and matrix properties for the calculations are provided in table 1.

To simplify the calculations and discussion, the two fibre types listed here are assumed to possess identical dimensions, scale parameter of breaking strain distribution, and the relative interface shear yielding stress $\tau_{\mathrm{y}}/E_{\mathrm{f}}$. Fibre type LE, however, has a tensile modulus 2.5 times greater than that of fibre type HE. Although our model is not targeted for any particular fibre types so that the conclusions will hold more generally, the LE fibre can be considered a carbon type, and the HE one belongs to the glass fibre family.

(a) *The distributions of the in situ fibre bundle breaking strains*

The distributions of the breaking strains for fibre bundles LE and HE can be derived using equations (3.4)–(3.6) when the parameters in table 1 are used. Those

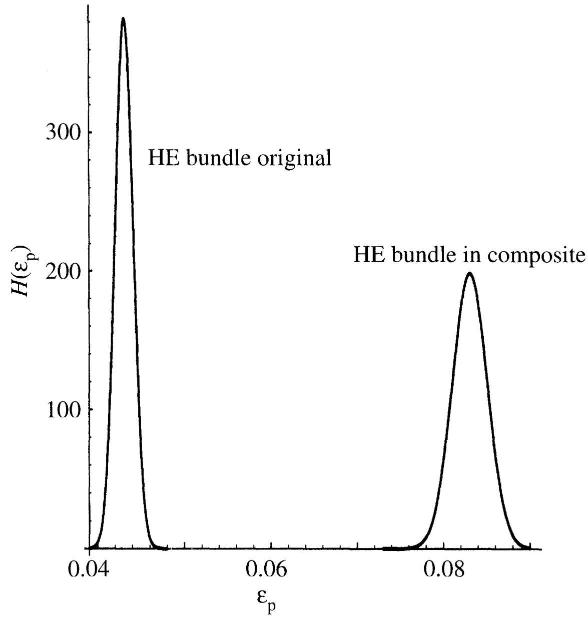


Figure 1. The original and *in situ* distributions of breaking strain for fibre bundle type HE.

fibre properties, however, will change due to the fibre–matrix interaction once the fibres are embedded into the matrix. According to the Weibull rule, the fragmentation process in a composite will generate a synergistic effect, leading to a higher fibre breaking strain because of the shorter length of the fibre fragments. Since the fibre tensile modulus remains constant up to failure, the strengths of the fibre segments will also increase owing to their direct relation to breaking strains. Therefore, although the distributions of the *in situ* fibre bundle breaking strains can still be found from equations (3.4)–(3.6), the original fibre length l_f in the equations has to be replaced by the critical fibre length l_c defined in equation (4.2) to reflect the fibre–matrix interaction.

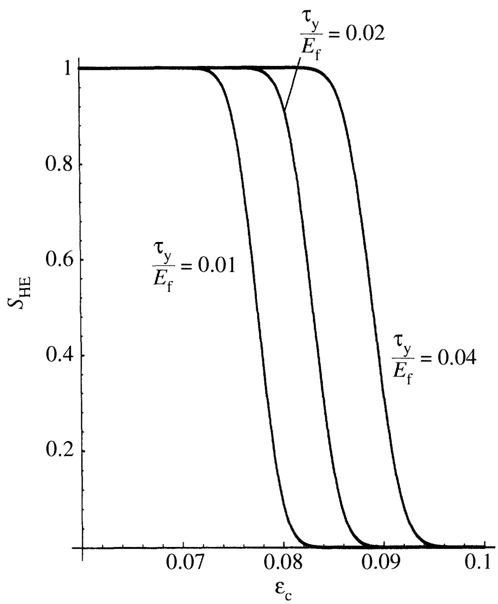
Figure 1 is thus plotted for the fibre bundle type HE. The curve on the left represents the original breaking strain distribution of the fibre bundle, and the curve on the right is the *in situ* distribution. Although both are normal forms, the *in situ* distribution has a higher mean value and a higher variation due to shorter length l_c.

(*b*) *The surviving fibre ratio S_i and the related parameters*

Based on equations (2.3), (3.4)–(3.6) and (4.2), we can show the effects of all relevant factors on the value of the surviving fibre ratio S_i. For simplicity, however, only the effect of the relative shear yielding stress τ_y/E_f on S_{HL} is provided in figure 2. In the figure, the S_{HL} value reduces, or more fibres break, as the composite strain ϵ_c increases. Evidently this depiction is closer to the actual case. Further, the interface strength is critical, and a higher shear yielding strength τ_y of the interface or a better fibre–matrix bonding will considerably alleviate the fibre breakage.

(*c*) *Prediction of the tensile modulus of hybrid composite*

Figure 3 illustrates the relationship of the composite tensile modulus on a relative scale against the composite strain ϵ_c according to equation (2.4). Note that although the composite tensile modulus ratio E_c/E_m is not explicitly shown in the equation



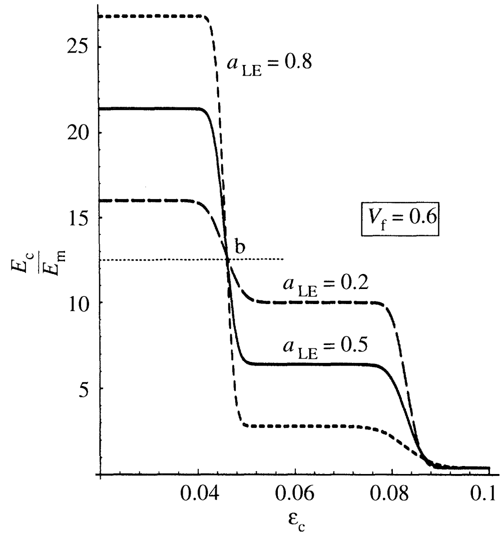


Figure 2. Effects of τ_y/E_f on the fibre surviving ratio S_{HE}.

Figure 3. Dependence of E_c/E_m on ϵ_c as well as on a_{LE}.

as a function of ϵ_c, the composite strain affects E_c/E_m considerably because of the S value defined in equation (2.3) which relates E_c/E_m to ϵ_c.

There are three curves in figure 3 corresponding to three levels of the fibre LE fraction a_{LE}; a higher a_{LE} value means more LE fibres of higher tensile modulus in the composite hence yielding a higher composite modulus before all LE fibres fail, and a lower composite modulus after their failure. It is worth mentioning that there is one point b in the figure where all curves merge. This is the point where the composite modulus becomes independent of fibre fraction a_{LE} although some of the LE fibres are still contributing towards composite properties and the composite is still a hybrid one.

Next, it is seen in figure 3 that at a given level of the fraction for fibre type LE, initially, the composite tensile modulus retains its highest value until the breakage of fibre type LE starts (when the composite strain level ϵ_c enters the range of the *in situ* breaking strains of the fibre LE). As the breakage of all fibre type LE is completed over a range of the composite strain, the composite modulus is reduced abruptly to a much lower level associated with the remaining fibre type HE. When the composite strain further increases, reaching the breaking range of fibre HE, a gradual breakage of fibre HE is also taking place and is reflected by the change of the composite modulus. The ultimate composite tensile modulus ratio E_c/E_m approaches 1 after all fibres fail and only the matrix, which has been assumed to have a higher breaking strain, carries the load.

(*d*) *Prediction of stress–strain curve of the hybrid composite based on three models*

It has been shown so far that there are two kinds of variations in fibre breaking strains. One type is the variation of breaking strain along a fibre length, and can be called the within-fibre variation. The second kind is the variation of breaking strains

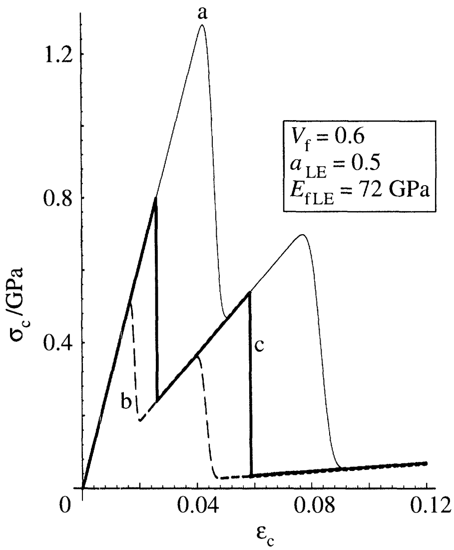


Figure 4. The relative composite stress–strain curves predicted using three models.

between fibres of the same type and can be termed as the between-fibre variation. Here obviously we do not consider the breaking strain difference between LE and HE fibre types as a variation in the statistical sense. Furthermore, it is apparent that the within-fibre variation determines the critical length l_c in the fragmentation process so as to affect the result of the fibre–matrix interaction. The between-fibre variation relates to the bandwidth of the spectrum of fibre breakage strain, the surviving fibre ratio S, or the breakage abruptness of the fibres of the same type.

Figure 4 is constructed using equation (2.2), with three curves: curve a is plotted using equation (2.2) which includes the effects due to the within-fibre variation reflected by the fragmentation process or the critical fibre length l_c defined in equation (4.2), and the between-fibre variation characterized by the parameter S in equation (2.3), as well as the possible cross coupling interaction between the LE and HE fibres. Curve b is also drawn using equation (2.2), except that the original fibre length instead of the critical fibre length is applied in constructing the fibre breaking strain distribution function from equations (3.4)–(3.6) so that the within-fibre variation is ignored. Curve c is based on the classic rule of mixtures, equation (2.1), where both the within-fibre and between-fibre variations are excluded.

In fact it is easy to see that the three stress–strain curves correspond to different structures: curve a represents the behaviour of a real hybrid composite, curve b shows the response of a hybrid fibre bundle, and curve c reflects the behaviour of a system made of two single fibres of LE and HE types. The very sharp peaks in curve c become more rounded in curve b, a reflection of the gradual breakage of fibres.

7. A parametric study

As stated before, for convenience of discussion, we will refer to the first peak and its associated properties in the stress–strain curve of the hybrid composite as the principal quantities, and those of the second peak as the secondary ones.

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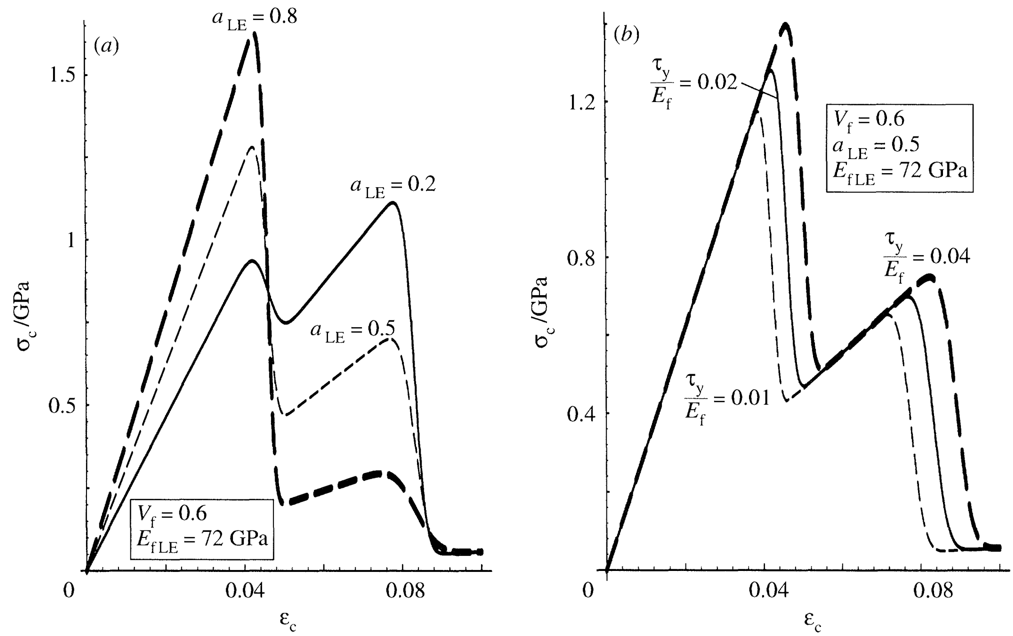

Figure 5. A parametric study on the composite stress–strain relationship. (*a*) The effects of a_{LE} on the composite stress–strain relationship. (*b*) The effects of $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ on the composite stress–strain relationship.

(*a*) *The stress–strain relationship of the hybrid composite*

Besides the within and between-fibre variations as well as the possible cross coupling effect between fibres of different types, there are other factors which will influence the composite behaviour, as illustrated in figure 3 which demonstrates how some of the factors affect the composite modulus.

First, let us examine the influences of the fibre fractions. Figure 5*a* illustrates the composite stress–strain curve at three different a_{LE} levels. The general trend is very similar to the modulus case in figure 3.

The effects of the relative shear yielding stress $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ on the composite stress–strain relation are depicted in figure 5*b*. A higher $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ value, or a better fibre–matrix bonding, increases the breaking strain and strength of the hybrid composite for both principal and secondary peaks.

Other conclusions can also be drawn from the above figures. For instance, comparing with the composite strength which is a function of many system and fibre parameters, composite breaking strain is more of an intrinsic and stable property. It hence confirms the advantage of using breaking strain to define the hybrid effect. Moreover, although figure 5 illustrate clearly how the composite stress–strain relation is influenced by the relevant factors, it doesn't provide an effective means to study the hybrid effect.

(*b*) *The anatomy of the hybrid effect*

From figures 4 and 5, we have learnt the effects of the important factors on the shape of the stress–strain relationship of a hybrid composite. We will next examine quantitatively how these factors may influence the hybrid effect.

As mentioned previously, there are two definitions for the hybrid effect. One defines the hybrid effect as the deviation from the rule of mixtures, and another focuses on the difference of the behaviour of a fibre in a monolithic and a hybrid composites. In other words, hybrid effect is revealed through comparison of different cases. Furthermore, few researchers in this field would ordinarily make distinction between a fibre and a fibre bundle when using the rule of mixtures. Having shown in the preceding section the discrepancy between the fibre and fibre bundles properties, it is clear that using fibre or fibre bundle into the rule of mixtures will yield drastically diverse results.

Therefore, depending on the situations, the hybrid effect defined as the deviation from the rule of mixtures may consist of contributions from several sources; the within-fibre property variation, the between-fibre variation and the cross coupling effect between the fibres of two different types. Whereas in the second definition comparing the fibre behaviour in a monolithic and a hybrid composites respectively, only the effect caused by the cross coupling between the fibres of different types in the hybrid composite is included, which is more of a genuine hybrid effect as it only exists in a hybrid composite. Therefore we can say that the deviation from the rule of mixtures is a necessary but not sufficient condition for the occurrence of the real hybrid effect, and the second definition is both necessary and sufficient.

In order to thoroughly investigate the hybrid effect and the factors involved, we will study each of the different possible cases by comparing a hybrid composite with single fibres, with fibre bundles and finally, with a monolithic composite. All the results will shed light collectively to the nature and causes of the hybrid effect.

(i) *Comparison of a hybrid composite with single fibres*

As stated above, comparison of the properties of an individual fibre and the corresponding properties of the fibre in the hybrid composite will show the deviation of the hybrid composite behaviour from the rule of mixtures relative to the fibre. The subscripts 'c' and 'f' below represent the composite and the fibres, respectively; and 1 and 2 correspond to the principal and the secondary peaks in the stress–strain curve of the hybrid composite.

The breaking strain of the composite is determined from equation (5.2), and the fibre breaking strain can either be its mean value from equation (3.2) or its mode as used by Pan (1995). Since the former is much more commonly used in practice, it is adopted here.

The difference between ϵ_{c1} and ϵ_{fLE}, as well as between ϵ_{c2} and ϵ_{fHE} in figure 6, is apparently due to the *in situ* interactions in the composite and is shown in the figure as function of the interface shear yielding strength τ_y. The short dotted line in the figure indicates a point where the composite breaking strain is equal to that of the individual fibre so that no hybrid effect, with reference to the fibre, exists. Below or above the line, the composite breaking strain is either lower or greater than that of the fibre, i.e. a negative or a positive hybrid effect is observed.

It is seen here that when the interface shear yielding strength τ_y is very weak, and the mechanism resulting from the fragmentation process becomes negligible, the ratio between the breaking strains of the composite and the fibre is smaller than 1. In other words, the composite breaking strain is lower than that of the fibre so that there is a negative hybrid effect caused by the gradual breakage of fibres in the composite due to the between-fibre variation. However, when the interface strengthens, the fragmentation process is closer to the saturation state so that the fibre fragments get shorter, with higher breaking elongation and hence higher strength. As a result, the

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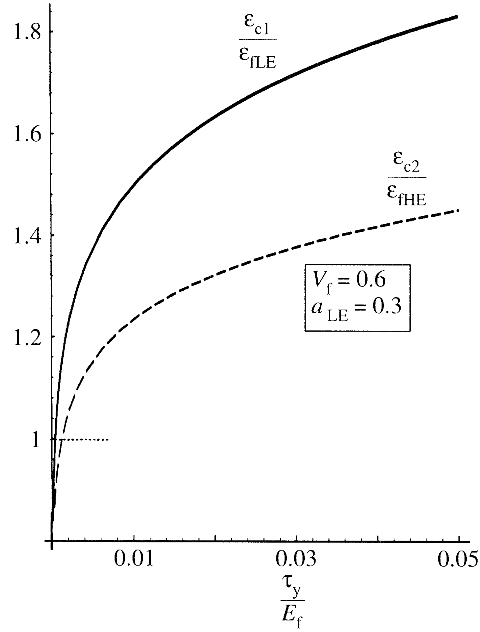

Figure 6. $\epsilon_{c1}/\epsilon_{fLE}$ and $\epsilon_{c2}/\epsilon_{fHE}$ versus τ_y/E_f.

composite becomes more extensible and stronger. The ratio between the breaking strains of the composite and the individual fibre thus increases and exceeds unity, resulting in a positive hybrid effect.

In summary, when comparison is made between a hybrid composite and a single fibre, either a negative or a positive hybrid effect can occur, depending on the quality of the fibre–matrix interface in the composite.

(ii) *Comparison of a hybrid composite with a fibre bundle*

As stated above that it is more rational to consider fibres in a composite as a bundle rather than as individual fibres. The next comparison is hence made between a hybrid composite and parallel fibre bundles whose mean breaking strains are calculated from equation (3.5) and used here. The characteristic variables we are interested here include the principal and secondary breaking strains of the hybrid composite, relative to those of the corresponding fibre bundles, i.e. $\epsilon_{c1}/\epsilon_{pLE}$ and $\epsilon_{c2}/\epsilon_{pHE}$. The differences between the two are attributed to the interactions between the fibres and the matrix, and also possibly to the interactions between the LE and HE fibres in the composite. However, the effect of the between-fibre variation will be cancelled in these ratios.

In addition, the ratio $\epsilon_{c1}/\epsilon_{c2}$ is also examined below to provide additional information. Any change in this ratio will offer us insight from a unique angle about the hybrid composite behaviour.

In the following discussion, figure 7 is focused on the principal strain ratio $\epsilon_{c1}/\epsilon_{pLE}$, and figure 8 on the secondary strain ratio $\epsilon_{c2}/\epsilon_{pHE}$, as indicators of the hybrid effects in relation to fibre bundles. It is seen from figures 7 and 8 that $\epsilon_{c1}/\epsilon_{pLE} > \epsilon_{c2}/\epsilon_{pHE} > 1$. In other words, since the comparison here is between a composite and a fibre bundle, the effect of the gradual fibre breakage due to the between-fibre variation is basically eliminated. Therefore, negative hybrid effects are non-existent and there

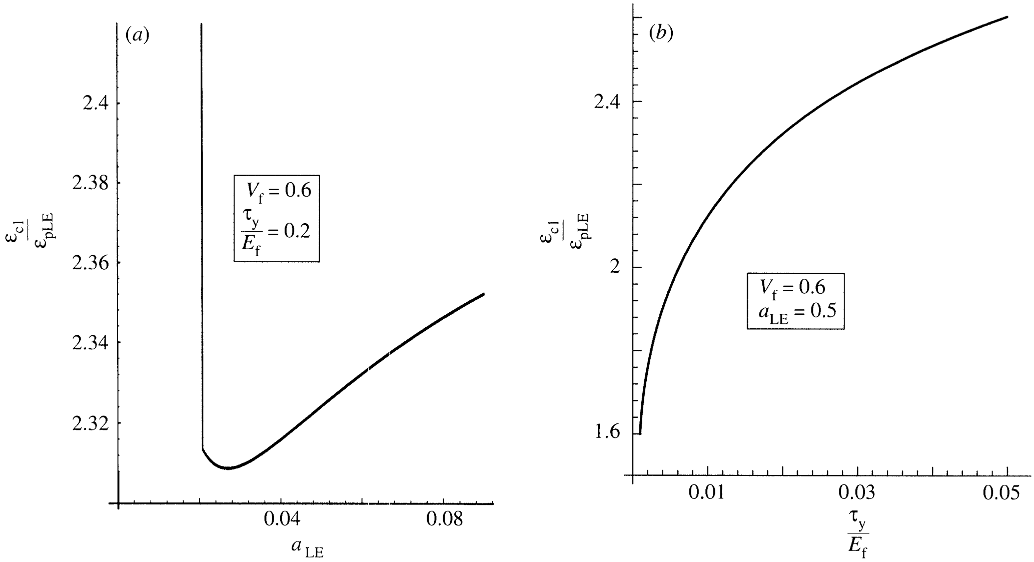

Figure 7. Comparisons between ϵ_{c1} and ϵ_{pLE}: (*a*) $\epsilon_{c1}/\epsilon_{pLE}$ versus the hybrid ratio a_{LE}. (*b*) $\epsilon_{c1}/\epsilon_{pLE}$ versus the relative interface yielding shear strength τ_y/E_f.

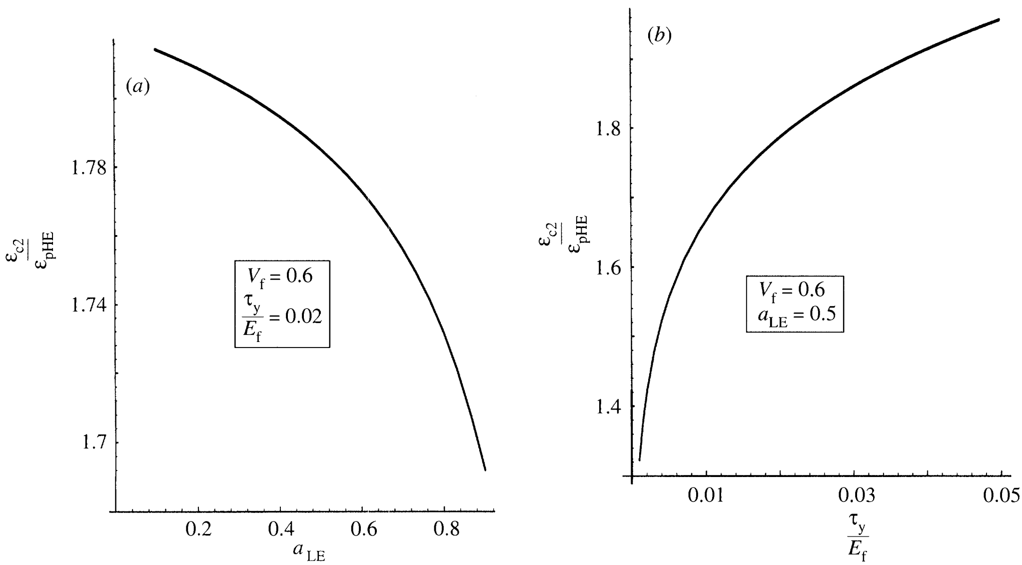

Figure 8. Comparisons between ϵ_{c2} and ϵ_{pHE}: (*a*) $\epsilon_{c2}/\epsilon_{pHE}$ versus the hybrid ratio a_{LE}. (*b*) $\epsilon_{c2}/\epsilon_{pHE}$ versus the relative interface yielding shear strength τ_y/E_f.

is always a positive hybrid effect in both the principal and the secondary breaking strains, but the magnitude of the hybrid effect is greater for the ratio $\epsilon_{c1}/\epsilon_{pLE}$.

Figure 9 shows the ratio $\epsilon_{c1}/\epsilon_{c2}$ of the principal and the secondary breaking strains. Obviously, there is always $\epsilon_{c1}/\epsilon_{c2} \leqslant 1$ or $\epsilon_{c1} \leqslant \epsilon_{c2}$.

A complete parametric study has been done through these figures. In figure 7*a*, ratio $\epsilon_{c1}/\epsilon_{pLE}$ is plotted against the hybrid ratio a_{LE} of the LE fibre type in the composite. Note that, since the bundle strain ϵ_{pLE} is not related to a_{LE}, the relationship between $\epsilon_{c1}/\epsilon_{pLE}$ and a_{LE} is proportional to the relation ship between ϵ_{c1} and a_{LE}.

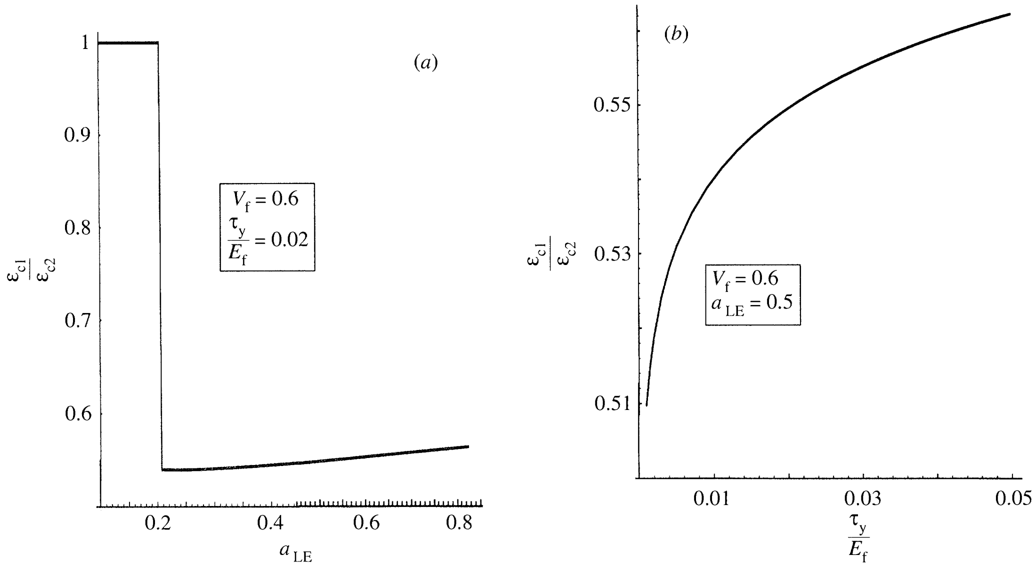

Figure 9. Comparisons between ϵ_{c1} and ϵ_{c2}: (*a*) $\epsilon_{c1}/\epsilon_{c2}$ versus the hybrid ratio a_{LE}. (*b*). $\epsilon_{c1}/\epsilon_{c2}$ versus the relative interface yielding shear strength τ_y/E_f.

From figure 7*a*, it is seen that a change of the relative proportion a_{LE} of the LE fibre will alter the principal breaking strain of the hybrid composite, but the relation is not monotonic. There is a critical value of a_{LE} which leads to the minimum value of $\epsilon_{c1}/\epsilon_{pLE}$; other values of a_{LE} will increase the principal breaking strain of the composite. However, when a_{LE} is below the critical point, the positive hybrid or the synergistic effect is at its highest.

A different conclusion can be drawn about the secondary breaking strain ratio $\epsilon_{c2}/\epsilon_{pHE}$ as shown in figure 8*a*. It is easy to understand that this strain ratio is also influenced by a_{LE} level because of the relation $a_{LE} + a_{HE} = 1$. Changing a_{LE} will alter a_{HE} value, the concentration in the system of the HE fibre type. This reveals at least one of the mechanisms of the mutual coupling interactions between the two fibre types in a hybrid composite.

When a_{LE} is at minimum, ϵ_{c2} possesses the highest value. As a_{LE} increases, the ϵ_{c2} will decrease monotonically.

The relation between a_{LE} and the ratio $\epsilon_{c1}/\epsilon_{c2}$ is shown in figure 9*a*. When a_{LE} is equal to zero, the composite becomes a monolithic one made of HE fibre only, so that $\epsilon_{c1} = \epsilon_{c2}$. However, when a_{LE} rises to a critical level, the single peak on the compo site stress–strain curve will split into two, yielding the principal breaking strain ϵ_{c1} and the secondary one ϵ_{c2}, and the ratio $\epsilon_{c1}/\epsilon_{c2}$ will then become smaller than 1, assuming its minimum level at this point. It is interesting to note that this ratio does not remain constant, and is affected by many factors, including a_{LE}. As a_{LE} keeps increasing, the ratio $\epsilon_{c1}/\epsilon_{c2}$ will increase, eventually returns to 1 as expected when $a_{LE} = 1$.

Moreover, the influence of a_{LE} on the two peaks corresponding to LE and HE fibre types is different in magnitude; when a_{LE} changes from 0.1 to 0.9, there is a 13.75% reduction for $\epsilon_{c2}/\epsilon_{pHE}$, and only a 4.76% reduction for $\epsilon_{c1}/\epsilon_{pLE}$. That is, a_{LE} has a higher impact on the HE bundle type, resulting in approximately 50% reduction for $\epsilon_{c1}/\epsilon_{c2}$.

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The influence of the relative shear yielding strength of the fibre–matrix interface τ_y/E_f on the three breaking strain ratios are illustrated in figures 7*b*, 8*b* and 9*b* respectively. Since a higher τ_y value represents a stronger fibre–matrix interface, and consequently the fragmentation process will get closer to saturation state, so that both $\epsilon_{c1}/\epsilon_{pLE}$ in figure 7*b* and $\epsilon_{c2}/\epsilon_{pHE}$ in figure 8*b* increase with τ_y. In other words, a stronger interface will result in a composite with higher breaking strains. The ratio $\epsilon_{c1}/\epsilon_{c2}$ in figure 9*b* also increases when τ_y rises, indicating that the effect of fibre–matrix interaction is more significant on the less extensible LE fibre type. It should be pointed out that, compared with the effect of a_{LE}, τ_y/E_f has a much greater influence on all these breaking strain ratios.

It is important to stress again that when comparing composite to fibre bundles, only positive hybrid effects can be found.

(iii) *Comparison of a hybrid with a monolithic composite*

It is most interesting to compare the hybrid composite to a non-hybrid composite made from either of the two fibre types. In this comparison, both influences associated with the within-fibre and between-fibre variations are eliminated so that the differences between the two composite types are entirely attributed to the interactions between LE and HE fibres in the hybrid composite. This will therefore provide information for direct examination of the cross coupling effects between the two fibre types in a hybrid composite.

Let us compare a hybrid composite, and a monolithic one made of the LE and the HE fibre bundle, respectively. The composites would otherwise be identical with the same fibre volume fraction V_f. The following figures show the effects of the important properties on the ratios $\epsilon_{c1}/\epsilon_{cLE}$ and $\epsilon_{c2}/\epsilon_{cHE}$, where ϵ_{cLE} and ϵ_{cHE} are the breaking strains, obtained using the same technique as in equation (5.2), of the monolithic composites made of the single fibre bundle type LE and HE, respectively.

First, it can be seen from figures 10 and 11 that, $\epsilon_{c1}/\epsilon_{cLE} > 1$ when a_{LE} is small, and $\epsilon_{c2}/\epsilon_{cHE} < 1$, indicating that the associated hybrid effect can be positive for the principal breaking strain, and always negative for the secondary breaking strain. Note that attention has to be paid below to the value of a_{LE} when dealing with the principal ratio $\epsilon_{c1}/\epsilon_{cLE}$.

It is deduced from figure 10*a* that, in the practical range of the relative hybrid ratio a_{LE}, the principal breaking strain ϵ_{c1} of the hybrid composite can be either higher or lower than the breaking strain ϵ_{cLE} of the monolithic composite, the latter being obviously independent of the a_{LE} value. That is, the hybrid effect of the breaking strain can be either positive or negative, depending on the levels of the variables involved. The ratio $\epsilon_{c1}/\epsilon_{cLE}$ is greater than 1, and is at its maximum level when a_{LE} is below a minimum value determined by the τ_y level, and will drop sharply to the value smaller than 1 once a_{LE} exceeds the minimum value; $\epsilon_{c1}/\epsilon_{cLE}$ will approach 1 as a_{LE} of the hybrid composite increases to 1 so that the hybrid composite evolves into a monolithic one made of LE fibre alone. This trend is consistent with that reported in Fukuda *et al.* (1984*a*), that is, the hybrid effect in breaking strain is positive when a_{LE} is very small, and becomes greater as a_{LE} further decreases. However, for relatively large a_{LE} values, the hybrid effect stays negative, and is almost independent of the level of τ_y/E_f.

Furthermore, as seen from figure 11*a*, the proportion of the LE fibre, a_{LE}, will also affect the ratio $\epsilon_{c2}/\epsilon_{cHE}$ for the reason mentioned previously in the case of fibre bundle. Initially when $a_{LE} \to 0$, so that the hybrid composite approaches a one made

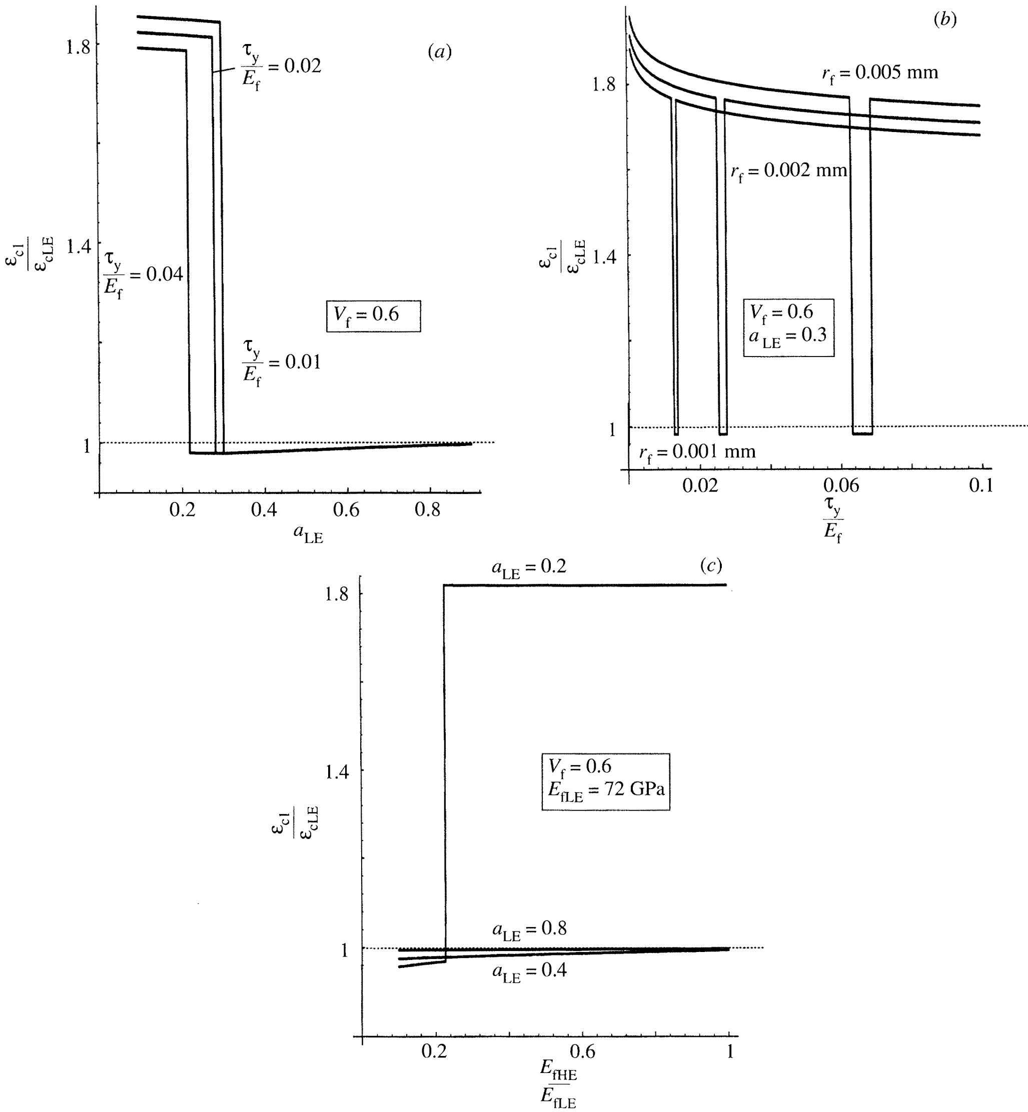

Figure 10. Comparisons between ϵ_{c1} and ϵ_{cLE}: (*a*) $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ versus the hybrid ratio a_{LE}. (*b*) $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ versus the relative interface yielding shear strength τ_y/E_f. (*c*) $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ versus the fibre modulus ratio $E_{\mathrm{fHE}}/E_{\mathrm{fLE}}$.

of HE fibre only, $\epsilon_{\mathrm{c2}}/\epsilon_{\mathrm{cHE}} = 1$. Increasing the a_{LE} value leads to a decreasing $\epsilon_{\mathrm{c2}}/\epsilon_{\mathrm{cHE}}$ value. Additionally, it is seen in the figure that the relative interface strength τ_y/E_f has little effect on the results since the curves at the three τ_y/E_f levels coincide with each other; this conclusion remains valid when the total fibre volume fraction V_f is not sufficiently large; this will be further explained in figure 11*b*.

The influence of τ_y on the two composites is more complex as depicted in figures 10*b* and 11*b*. In figure 10*b*, the ratio $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ is mostly greater than 1, but this ratio decreases slightly as τ_y/E_f increases, except for certain sharply defined ranges of τ_y/E_f where $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ becomes less than 1 and appears independent of τ_y/E_f. For a thicker fibre, this range of τ_y/E_f is wider and located at the higher end of τ_y/E_f axis.

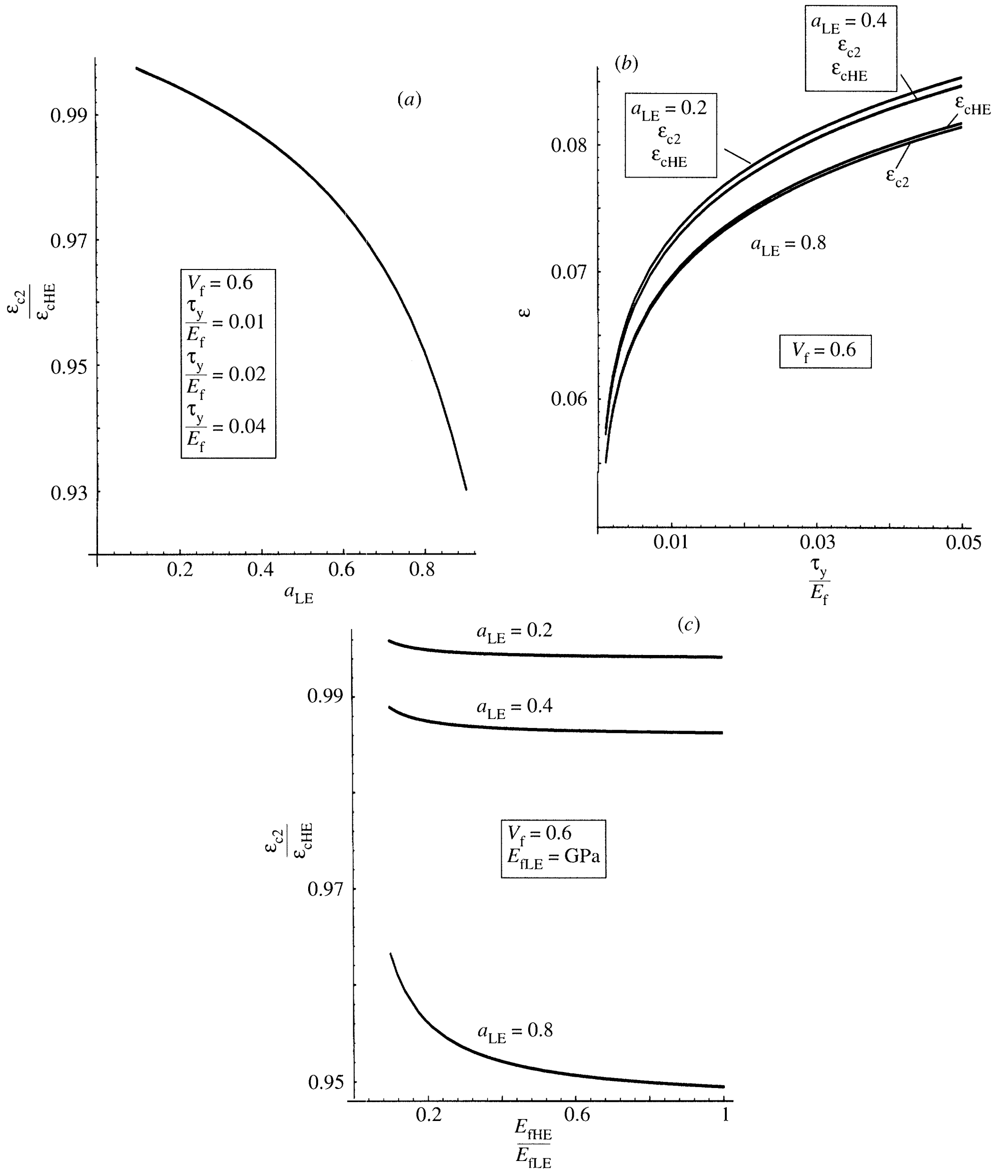

Figure 11. Comparisons between ϵ_{c2} and ϵ_{cHE}: (*a*) $\epsilon_{c2}/\epsilon_{cHE}$ versus the hybrid ratio a_{LE}. (*b*) $\epsilon_{c2}/\epsilon_{cHE}$ versus the relative interface yielding shear strength τ_y/E_f. (*c*) $\epsilon_{c2}/\epsilon_{cHE}$ versus the fibre modulus ratio E_{fHE}/E_{fLE}.

The secondary case in figure 11*b* is also intricate. First, we attempted to plot the ratio $\epsilon_{c2}/\epsilon_{cHE}$ against τ_y/E_f at different r_f levels. The results for different r_f values turned out to be completely coincide with each other. In other words, the fibre radius in this case shows no net influence on the ratio $\epsilon_{c2}/\epsilon_{cHE}$. We then tried to replace it with a plot of $\epsilon_{c2}/\epsilon_{cHE}$ versus τ_y/E_f at three levels of a_{LE}, only to find out that it was impossible to obtain smooth curves because of severe oscillations encountered during calculation. Consequently, we plotted two separate curves corresponding to ϵ_{c2} and ϵ_{cHE} instead of their ratio, against τ_y/E_f at three levels of the relative hybrid ratio a_{LE}.

In figure 11*b*, when τ_y/E_f becomes larger, the values of both ϵ_{c2} and ϵ_{cHE} increase. However, the influence of τ_y/E_f on the hybrid effect is dependent on the level of the relative hybrid ratio a_{LE}. When a_{LE} is small, say equal to 0.2 and 0.4 as in the figure, there will be no noticeable difference between ϵ_{c2} and ϵ_{cHE} and the two curves coincide, so that the ratio $\epsilon_{c2}/\epsilon_{cHE} \rightarrow 1$. Only when a_{LE} is very large ($a_{LE} = 0.8$), will there be a tiny difference between ϵ_{c2} and ϵ_{cHE}. The ratio $\epsilon_{c2}/\epsilon_{cHE}$ becomes smaller than 1 and a negative hybrid effect emerges; this ratio further reduces slightly with τ_y/E_f. Therefore, it can be concluded that τ_y/E_f has a little influence on the difference between ϵ_{c2} and ϵ_{cHE}, and a negative hybrid effect can be seen only when a_{LE} is extremely high. This explains why in figure 11*a* the curves corresponding to the three levels of τ_y/E_f coincide when a_{LE} is not sufficiently large ($a_{LE} = 0.6$ there).

Currently, we are working on a revised algorithm where the ratio τ_y/E_f will be differentiated between LE and HE fibre types. Then we can alter the ratio for one fibre type to observe the response of another fibre type so as to determine whether the ratio τ_y/E_f causes any cross coupling effects between the two fibre types in a hybrid composite.

The influence of the fibre moduli on the hybrid effects is illustrated in figures 10*c* and 11*c* for the principal and secondary cases respectively. Here we again set $E_{fLE} = 72.0$ GPa, and increase E_{fHE} to change the modulus ratio E_{fHE}/E_{fLE}. Considering figure 10*c* first, the effect of the modulus ratio on ϵ_{c1} and ϵ_{cLE} is dependent on the level of the relative hybrid ratio a_{LE}. When a_{LE} is greater than a critical value, $\epsilon_{c1}/\epsilon_{cLE} \leqslant 1$, meaning that there is a small but negative hybrid effect which is almost independent of E_{fHE}/E_{fLE} value. When a_{LE} is smaller than the critical level, however, $\epsilon_{c1}/\epsilon_{cLE}$ is smaller than 1 initially when E_{fHE} is low, increases slightly as E_{fHE}/E_{fLE} becomes large, then jumps very sharply to a level higher than 1 as E_{fHE}/E_{fLE} exceeds a critical level, and remains constant as E_{fHE}/E_{fLE} further increases. In other words, the effect of E_{fHE}/E_{fLE} is more significant when a_{LE} is very small. This effect of E_{fHE} on $\epsilon_{c1}/\epsilon_{cLE}$ provides another piece of evidence for the existence of the cross coupling influence between LE and HE fibre types in the hybrid composite.

In figure 11*c*, as E_{fHE}/E_{fLE} becomes greater, the ratio $\epsilon_{c2}/\epsilon_{cHE}$ reduces. However, as seen in the figure, the influence of E_{fHE}/E_{fLE} on $\epsilon_{c2}/\epsilon_{cHE}$ becomes more significant for higher values of a_{LE}, meaning that the negative hybrid effect is less noticeable when a_{LE} is small, consistent with the result in figure 11*a*.

It has to be emphasized here that the causes of the hybrid effects revealed in this study are not complete since some of the factors such as the thermal stress, stress concentration, load sharing or fibre dispersion and the dynamic effects during fibre breakage have been ignored. However, it is reasonable to believe that the causes we have found from the present analysis responsible for the hybrid effects will still be valid and play major roles even in a more sophisticated study.

8. Conclusion

There are three physical mechanisms identified in this study which are responsible for causing deviations of behaviour of a hybrid composite from the prediction by the rule of mixtures. The first (mechanism I) is reflected through the fragmentation process due to the fibre–matrix interactions. This effect is largely dependent on the within-fibre property variation and the fibre–matrix interfacial shear strength. The second mechanism (mechanism II) is related to the between-fibre variation, i.e. variation of breaking strains between fibres of the same type. Fibres of the same type in the composite will therefore break gradually, eventually resulting in a lower breaking elongation and strength for the composite. In other words, mechanism I is the main factor causing a positive hybrid effect in both composite breaking stress and strain. Mechanism II is responsible for the occurrence of a negative hybrid effect. The third (mechanism III) is the cross coupling effects between the LE and HE fibres in a hybrid composite; this mechanism, in general, leads to a positive hybrid effect, at low a_{LE} level, associated with the principal peak, and a negative hybrid effect associated with the secondary peak, when compared with monolithic composites.

It is clear that the definition of hybrid effect as the deviation from the rule of mixtures in fact includes all the three mechanisms. But both mechanisms I and II exist even in a composite made of a single fibre type, and the effects associated with these two mechanisms will also occur in non-hybrid composites. Therefore, the second definition focusing on the difference of fibre behaviour in a hybrid and a monolithic composites is more rational since it only reflects the effect associated with the mechanism III. We can hence conclude from this study that the first definition is a necessary but not sufficient condition for the occurrence of the real hybrid effect, and the second definition is both necessary and sufficient.

The effects of the three mechanisms are also dependent on the key variables involved, including the relative hybrid ratio a_{LE} for fibre LE, system fibre volume fraction V_{f}, the relative shear yielding strength of the fibre–matrix interface $\tau_{\mathrm{y}}/E_{\mathrm{f}}$, and the fibre tensile modulus ratio $E_{\mathrm{fHE}}/E_{\mathrm{fLE}}$.

There could be three slightly different ways to define and study the hybrid effect, by comparing hybrid composites with single fibres, with fibre bundles, and with monolithic composites. The conclusions drawn from the different comparisons are diverse, and great attention has to be paid when applying these conclusions.

In case of comparison between a hybrid composite with individual fibres, there can be a positive or a negative hybrid effect for both principal and secondary breaking strains. When the interface between fibre and matrix is very poor, negative hybrid effects exist for both the principal and secondary breaking strains. By strengthening the interface sufficiently, however, the hybrid effects for the two breaking strains become increasingly positive.

When comparing the hybrid composite with fibre bundles, the hybrid effects of the breaking strain are always positive due to the elimination of the mechanism II, but the magnitude of the principal hybrid effect is greater than the secondary one. The hybrid ratio a_{LE} exhibits a critical level at which $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{pLE}}$ reaches its minimum, and $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{pLE}}$ will become maximum when a_{LE} is below that critical level. As a_{LE} exceeds the critical level, increasing a_{LE} will also increase $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{pLE}}$. On the other hand, there is a monotonic relation between a_{LE} and the hybrid effect in the secondary breaking strain; increasing a_{LE} will reduce the magnitude of $\epsilon_{\mathrm{c2}}/\epsilon_{\mathrm{pHE}}$. A greater $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ will enhance the hybrid effects in both the principal and secondary quantities.

We also examine the influence of the related variables on the ratio of the principal and secondary breaking strains $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{c2}}$. It is found that increasing $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ causes an increase in $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{c2}}$. Whereas for the hybrid ratio a_{LE}, its influence on $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{c2}}$ is not continuous.

We then compared the hybrid composite with a monolithic composite to demonstrate the existence of the cross coupling interactions between the LE and HE fibre types, and to study the nature of these interactions. It has been shown that the hybrid effects not only exist for the lower elongation LE fibres, but also for high elongation HE fibres.

Moreover, it has been proved in this study that the mutual coupling effects between the LE and HE fibres do exist in a hybrid composite. For example, changing the relative proportion of the LE fibre, a_{LE}, will influence the breaking strain of the secondary peak associated with HE fibre, while altering the tensile modulus E_{fHE} will affect the principal breaking strain.

In general, the ratio $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ at low a_{LE} level is greater than 1, and $\epsilon_{\mathrm{c2}}/\epsilon_{\mathrm{cHE}}$ is smaller than 1, meaning that there is a positive hybrid effect for the former and a negative hybrid effect for the latter. However, the hybrid effect will become negative for the principal breaking strain when a_{LE} is greater than a small critical level (this critical level is not necessarily identical to that for the previous fibre bundle case in figure 5*a*). On the other hand, a higher a_{LE} will further reduce the ratio $\epsilon_{\mathrm{c2}}/\epsilon_{\mathrm{cHE}}$.

Furthermore, although the ratio $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ is found mostly greater than 1 at low a_{LE} level, it decreases slightly when $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ increases, except for certain narrow ranges of $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ with sharp boundaries where $\epsilon_{\mathrm{c1}}/\epsilon_{\mathrm{cLE}}$ becomes less than 1 and then remains independent of $\tau_{\mathrm{y}}/E_{\mathrm{f}}$. The location and width of this range of $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ are related to the fibre radius: a thicker fibre corresponds to a wider range with higher $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ values. However, $\tau_{\mathrm{y}}/E_{\mathrm{f}}$ has very little influence on the secondary breaking strain ratio $\epsilon_{\mathrm{c2}}/\epsilon_{\mathrm{cHE}}$. There will be an extremely small negative hybrid effect between the two only when a_{LE} is unrealistically high.

The factors associated with the thermal stress, fibre dispersion, stress concentration and other variables related to the dynamic fracture process, which will also contribute to the interactions between the constituents, have been excluded in this study. This exclusion, however, should not affect the validity of the conclusions we have drawn from this analysis.

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